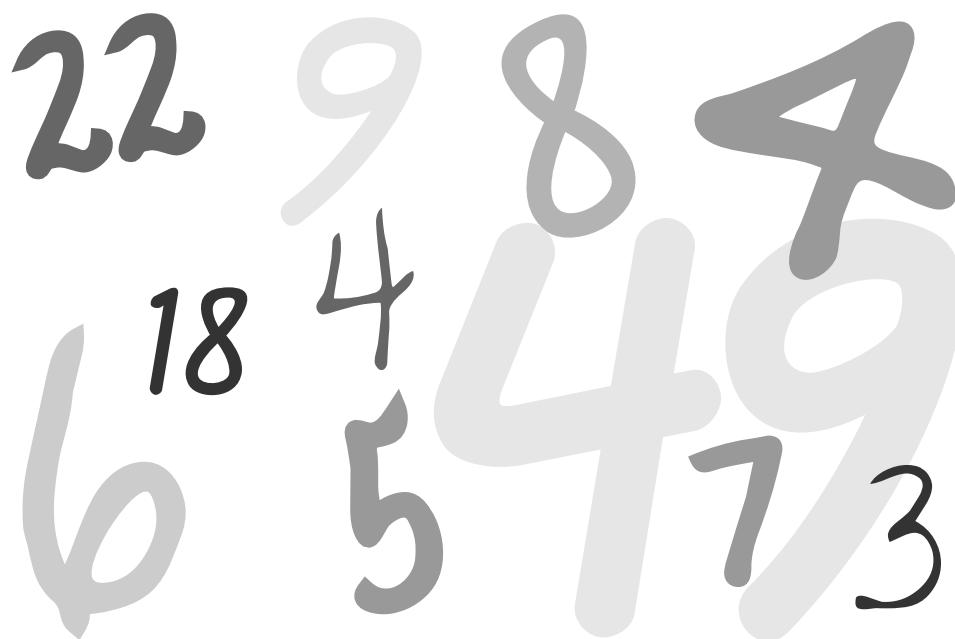


**Michigan Educational
Assessment Program
MEAP**



**High School
Mathematics Assessment
Model**

April 2001



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TABLE OF CONTENTS

INTRODUCTION	i
Purpose of This Document	i
Organization.....	i
PART ONE: FRAMEWORK OF THE ASSESSMENT	1
Vision for Mathematics	1
Comprehensive Assessment	2
Purpose of Statewide Assessment.....	2
Standards Assessed at the State Level.....	3
Standards for the Assessment Design.....	4
Item Types	5
Distribution of Items by Strand and Type of Format.....	6
Context for High School Item Selection.....	7
Sample Grade 11 Items	7
Assessment Reports.....	7
Teaching and Learning Standards.....	8
PART TWO: RELEASED TEST FORM	11
PART THREE: SCORING	39
Student Responses and Scoring.....	40
Answer Grid	70
PART FOUR: RESOURCES AND ACKNOWLEDGMENTS	71
Glossary.....	71
Mathematics Educational Resources.....	72
MEAP Mathematics Assessment Planning Committee	73
Mathematics Content Advisory Committee	74
Mathematics Bias Review Committee	76
MEAP High School Mathematics Test – Reference Sheet.....	79
MEAP Mathematics – Overlay Sheet.....	81

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INTRODUCTION

This document, the Mathematics Assessment Model, is designed to communicate the substance and format of the new MEAP mathematics assessment for 2001–2. Audiences for this assessment model include students, parents, teachers, curriculum specialists, and administrators. The model represents a collaborative effort under the joint leadership of the Michigan Department of Treasury’s Michigan Educational Assessment Program and the Michigan Department of Education’s Curriculum Development Program Unit. Major review and input was further provided by content advisory committee members who played a substantial role in the development of the new mathematics tests.

PURPOSE OF THIS DOCUMENT

The primary purpose of this document is to acquaint all teachers of mathematics with the design of the assessment. The mathematics assessment was developed from, and is aligned with, *the Mathematics Standards of the Michigan Curriculum Framework*. A review of the released test form in this document will give teachers concrete examples of assessment items. Because these items are released, they will not appear in operational forms of the assessments. Each of the items included in the Assessment Model was drawn from items that were tested with a representative sample of Michigan students at grade 11 during the Fall 2000 item tryout. All portions of this document may be copied and distributed for educational use.

ORGANIZATION

The Mathematics Assessment Model contains four parts.

Part one includes introductory material that details the number, type, context, and point value of the assessment items. This first part represents the basic framework on which the assessment was built.

Part two of the Assessment Model contains a released form of the assessment. This model is a complete test form which represents what students will be given at the time of testing.

Part three contains a sample of student responses, scoring guides, and annotations on scoring.

Part four contains a glossary, a brief list of resources, and the names of individuals who were instrumental in the development of this mathematics assessment in Michigan.

PART ONE: FRAMEWORK OF THE ASSESSMENT

VISION FOR MATHEMATICS

A mathematically powerful individual should be able to:

- *reason mathematically;*
- *communicate mathematically;*
- *problem solve using mathematics; and*
- *make connections within mathematics and between mathematics and other fields.*

Mathematics is the science of patterns and relationships. It is the language and logic of our technological world. Mathematical power is the ability to explore, to conjecture, to reason logically and to use a variety of mathematical methods effectively to solve problems. The ultimate goal of mathematics education is for all students to develop mathematical power to participate fully as citizens and workers in our contemporary world.

The 15 standards listed in the *Mathematics Content Standards of the Michigan Curriculum Framework* establish expectations for what students should know and be able to do as a result of their mathematics education.

The Mathematics Content Standards are grouped into six categories called strands, which are further divided into content standards. These standards, expressed as attributes we envision for our graduates, are the intended results of students' experiences with a core curriculum. Students are expected to make continuous progress toward meeting the standards at each level of schooling. All standards should be pursued at every grade level of a school's core curriculum from kindergarten through graduation. Benchmarks are established for each standard to designate what students are expected to learn within grades K-4, 5-8, and 9-12 to indicate progress toward meeting these standards. The assessment is designed from a subset of these benchmarks determined to be "testable" on a statewide assessment.

COMPREHENSIVE ASSESSMENT

A comprehensive assessment system consists of a statewide, district, and classroom assessment, and performance standards that form a link to content standards and teaching and learning standards. No single assessment instrument can provide all the information needed to accomplish the ultimate purpose of assessment which is to improve teaching and learning. An integrated approach that is necessary and includes both mathematics content standards and benchmarks, assessed with the most appropriate methods available.

PURPOSE OF STATEWIDE ASSESSMENT

The statewide mathematics assessment is designed to provide information about student achievement in mathematics, promote assessment practices that support learning for all students, and foster curriculum and instruction that is aligned with the state *Mathematics Content Standards*. This assessment may be used by students, parents, teachers, school districts, researchers, and state government to advance the quality of mathematics education.

STANDARDS ASSESSED AT THE STATE LEVEL

Statewide assessment is based on the *Content Standards for Mathematics of the Michigan Curriculum Framework*. Figure 1 presents the 15 content standards to be assessed.

Patterns, Relationships and Functions	Geometry and Measurement	Data Analysis and Statistics	Number Sense and Numeration	Numerical and Algebraic Operations and Analytical Thinking	Probability and Discrete Mathematics
I.1 <i>Patterns</i>	II.1 <i>Shape and Shape Relationships</i>	III.1 <i>Collection, Organization and Presentation Of Data</i>	IV.1 <i>Concepts and Properties Of Numbers</i>	V.1 <i>Operations and Their Properties</i>	VI.1 <i>Probability</i>
I.2 <i>Variability and Change</i>	II.2 <i>Position</i>	III.2 <i>Description and Interpretation</i>	IV.2 <i>Representa- tion and Uses Of Numbers</i>	V.2 <i>Algebraic and Analytical Thinking</i>	VI.2 <i>Discrete Mathematics</i>
	II.3 <i>Measurement</i>	III.3 <i>Inference and Prediction</i>	IV.3 <i>Number Relationships</i>		

Figure 1: Mathematical Standards to be Assessed

STANDARDS FOR THE ASSESSMENT DESIGN

Seven standards* were applied as criterion to guide the development of items for this assessment:

Organization of Information – The task asks students to organize, synthesize, interpret, explain, or evaluate complex information in addressing a concept, problem, or issue.

Consideration of Alternatives – The task asks students to consider alternative solutions, strategies, perspectives, or points of view in addressing a concept, problem, or issue.

Disciplinary Content – The task asks students to show understanding and/or use ideas, theories, or perspectives considered central to an academic or professional discipline.

Disciplinary Process – The task asks students to use methods of inquiry, research, or communication characteristics of an academic or professional discipline.

Communication – The task asks students to communicate their understandings, explanations or conclusions.

Problem Connected to the World Beyond the Classroom – The task asks students to address a concept, problem or issue similar to one they have encountered or are likely to encounter in life beyond the classroom.

Audience Beyond the School – The task asks students to communicate their knowledge, present a product or performance, or take some action for an audience beyond the teacher, classroom, and school building.

* These seven standards were adopted from the Center on Organization and Restructuring of Schools at the University of Wisconsin.

ITEM TYPES

The assessment features two types of items. Both require students to apply what they have learned and to analyze information presented by a prompt. The two types of items are:

- Selected response: Students select a response from four possible choices. These items can involve multiple steps and computations but generally require a short amount of time to complete.
- Constructed response: Students supply a solution to a problem to be evaluated in terms of both the final response and the approach used to reach that response. Multiple approaches to solving these problems are accepted and partial credit is granted for incomplete work. Effectively demonstrating one's work is critical. This work can be expressed in various forms, e.g., a narrative, calculations, charts, or graphs.

DISTRIBUTION OF ITEMS BY STRAND AND TYPE OF FORMAT

Figure 2 shows the distribution of test items by strand and item type. Selected-response items are worth 1 point and constructed-response items are worth 4 points. On the grade 11 test, 32 of the 52 points possible are assigned to selected-response items and 20 points assigned to constructed-response items. In other words, about 40% of the total score is attributed to constructed-response items.

Emphasis by Strand and Item-Type

STRAND	High School %
Patterns, Relationships, and Functions	20
Geometry and Measurement	20
Data Analysis and Statistics	20
Number Sense and Numeration	5
Numerical and Algebraic Operations and Analytical Thinking	20
Probability and Discrete Mathematics	15

ITEM TYPE	Items	Points	Percent Of Points
Selected-Response (1 point each)	32	32	62
Constructed-Response (4 points each)	5	20	38
Total Assessment	37	52	100

Figure 2: Distribution of Items Grade 11

CONTEXT FOR HIGH SCHOOL ITEM SELECTION

This test will address the “testable” portion of the state curriculum framework for high school. Furthermore, only partial coverage of the total set of testable benchmarks is provided for in any single assessment.

The assessable content is defined in “Assessable Content: Content Standards and Benchmarks for Mathematics,” which can be reviewed on the Merit Website: www.meritaward.state.mi.us.

SAMPLE GRADE 11 ITEMS

To help educators anticipate the kinds of items being planned for state assessments, this document includes a complete released form of this test for grade 11.

The *Mathematics Assessment Model* reflects a commitment to constructed response items because these types of items are necessary to determine the extent to which students are meeting the *Model Content Standards for Mathematics*. Examples of scoring guides that might be used to score these items follow the released form in Part Three of this document.

ASSESSMENT REPORTS

The results of the mathematics assessments will be reported by the strands of the *Content Standards*. The following strands, or combination of strands, will be reported on the High School Test :

- Patterns, Relationships, and Functions
- Geometry and Measurement
- Data Analysis and Statistics
- Number Sense and Numeration combined with Numerical and Algebraic Operations and Analytical Thinking
- Probability and Discrete Mathematics

TEACHING AND LEARNING STANDARDS

The connection between instruction and assessment is an important link in designing an assessment. The following instructional standards were considered in the design of the assessment.¹

Standard 1: Higher Order Thinking

Instruction involves students in manipulating information and ideas by synthesizing, generalizing, explaining, hypothesizing, or arriving at conclusions that produce new meanings and understandings for them.

Higher order thinking requires students to manipulate information and ideas in ways that transform their meaning and implications. This occurs when students combine facts and ideas in order to synthesize, generalize, explain, hypothesize or arrive at some conclusion of interpretation. Manipulating information and ideas through these processes allows students to solve problems and discover new meanings (for them) and understandings.

Lower order thinking occurs when students are asked to receive or recite factual information or to employ rules and algorithms through repetitive routines. As information receivers, students are given pre-specified knowledge ranging from simple facts and information to more complex concepts. Students are not required to do much intellectual work since the purpose of instruction is simply to transmit knowledge or to practice procedural routines.

Standard 2: Deep Knowledge

Instruction addresses central ideas of a topic or discipline with enough thoroughness to explore connections and relationships and to produce relatively complex understandings.

Knowledge is deep when central ideas of a topic or discipline are explored in considerable detail that shows interconnections and relationships. Knowledge is deep when, instead of being able to recite only fragmented pieces of information, students express relatively systematic, integrated or holistic understandings of central concepts. Mastery is demonstrated by students discussing relationships, solving problems, constructing explanations, and drawing conclusions.

Depth of knowledge and understanding can be indicated by the substantive character of the ideas that the teachers present in the lesson and by the level of understanding students demonstrate as they consider these ideas.

Standard 3: Substantive Conversation

Students engage in extended conversational exchanges with the teacher and/or with their peers about subject matter in a way that builds an improved and shared understanding of ideas or topics.

In classes characterized by high levels of substantive conversation there is sustained teacher-student and/or sustained student-student interaction about a topic. The interaction is reciprocal and it promotes coherent shared understanding. Substantive conversation has three features:

1. The talk is about subject matter in the discipline and includes higher order thinking such as making distinctions, applying ideas, forming generalizations, raising questions, not just reporting of experiences, facts, definitions, or procedures.
2. The conversation involves sharing of ideas and is not completely scripted or controlled by one party (as in teacher led recitation). Sharing is best illustrated when participants explain themselves or ask questions in complete sentences, and when they respond directly to comments of previous speakers.
3. The dialogue builds coherently on participants' ideas to promote improved, collective understanding of a theme or topic.

To recognize substantive conversation, we first define an interchange as a statement by one person and a response by another. The interchanges need not be between the same two people, but they must be linked substantively as consecutive responses.

Standard 4: Connections to the World Beyond the Classroom

Students make connections between substantive knowledge and either public problems or personal experiences.

A lesson gains in authenticity the more there is a connection to the larger social context in which students live. There are at least three ways in which student activity in classrooms can reflect some connections to life beyond school. First, lessons might focus on understanding a real-world, public problem of some contemporary significance; for example, applying statistical analysis in preparing a report to the city council on the homeless. Second, lessons can build upon students' personal experiences to connect to important ideas in the disciplines; for example, by comparing approaches to conflict resolution between people and nations. Finally, if students attempt to communicate their knowledge to others beyond the classroom, to influence or to assist others, school knowledge is more likely to have value beyond simply achieving success in school.

¹ *A Guide to Authentic Instruction and Assessment: Vision, Standards and Scoring* (1995) written by Fred M. Newmann, Walter G. Secada, and Gary G. Wehlage at the Wisconsin Center for Education Research.

PART TWO: RELEASED TEST FORM



GRADE 11 MATHEMATICS ASSESSMENT MODEL Spring 2001

SAMPLE GRADE 11 ITEMS

To help educators anticipate the kinds of items being planned for state assessments, this document includes a complete released form of this test for grade 11. *While this form may be administered to students as a practice test, it should not be assumed to duplicate precisely the difficulty levels of any future live forms of the test.*

Directions

Day One: Mathematics Assessment

Directions to the Student

In this test you will demonstrate your understanding of mathematics. The test has two parts. You will have at least 55 minutes to finish each part of the test. You will be given additional time if necessary.

You must record your answers to **all** questions in your TEST BOOKLET.

Use only a No. 2 pencil to mark your answers. Circle the correct letter completely. If you erase an answer, be sure to erase the first circled letter completely. You may use calculators on this test.

You may use the MEAP Reference Sheet as you take this mathematics test. It includes formulas, diagrams, and tables that you may find useful.

There are two types of questions on this test:

1. Some questions will require you to choose the best answer from among four answer choices.
2. Some questions will require you to write, explain, or show your work in the space provided in your test booklet.

Here are some important things to remember as you take this test:

1. Read each question carefully and think about the answer.
2. If answer choices are given, choose the best answer by circling the letter in your test booklet.
3. If an answer space is provided in your test booklet, show all your work and write your answer neatly and clearly in the space provided.
4. You should have plenty of time to finish every question on the test. If you do not know the answer to a question, go on to the next question. You may return to that question later.
5. If you finish early, you may check your work in that part of the test **only**. Do **not** look at the questions in other parts of the test.
6. When you reach the word **STOP** in your test booklet, do **not** turn the page.

Once you have finished each part, close your test booklet and put down your pencil.

If you do not understand any of these directions, please raise your hand.

Sample Items

Sample Questions

To help you understand the test questions, look at the sample test questions that follow. These samples will show you what the questions in the test are like and how to mark your answers.

Multiple-Choice Sample Question

For this type of question, you will determine the answer and circle the correct letter in your test booklet.

- S1** In a recent public service message, a rock star announced that a person loses an average of 7 minutes of life for every cigarette the person smokes. If a person smokes 20 cigarettes (one pack) a day for the next 10 years, by approximately how many years will his or her life be shortened?
(365 days = 1 year)

- A** $\frac{1}{2}$ year
- B** 1 year
- C** 7 years
- D** 10 years

For this sample question, the correct answer is **B**.

Sample Items

Open-ended Sample Question

For this type of question, you will write, explain, or show all your work in the space provided in your test booklet.

- S2** Jasmine is doing pushups as part of her exercise program. She did 2 the first day, 3 the second day, and 5 the third day. Each day she wants to do as many pushups as she did on the previous two days combined.
- A** Based on the information above, how many pushups would she have to do the sixth day?
Extend the pattern through day six to support your answer.

Day 3 Day 6
Pattern 2, 3, 5, 8, 13, 21

Jasmine would have to do 21 pushups the sixth day.

- B** Is it realistic for Jasmine to continue this program for pushups? Explain your answer. You may extend the pattern further to support your explanation.

Day 5 Day 6 Day 10 Day 12
Pattern 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377

No, because as the days continue, the combining of the previous two days' number of pushups will start to become excessively large for the average person to do.

*For example: Day 10 would be 144 pushups and
Day 12 would be 377 pushups if the
pattern continued.*

For this sample question, you would answer Part A by explaining that Jasmine would have to do 21 pushups the sixth day and show your work. For Part B you would answer no, because as the days continue, the combining of the previous two days' number of pushups will start to become excessively large for the average person to do. Remember to show your work.

High School—Part I

- 1** The minute hand on a clock pointed directly at the 12. It then moved 2700° clockwise. At which number is the minute hand pointed after this rotation?
- A** 3
B 6
C 10
D 12
- 2** Melody Music Club sells compact discs to club members on the Internet. This month it offers four different pricing packages. If members pay dues and buy exactly 5 CDs, which package offers the best value?

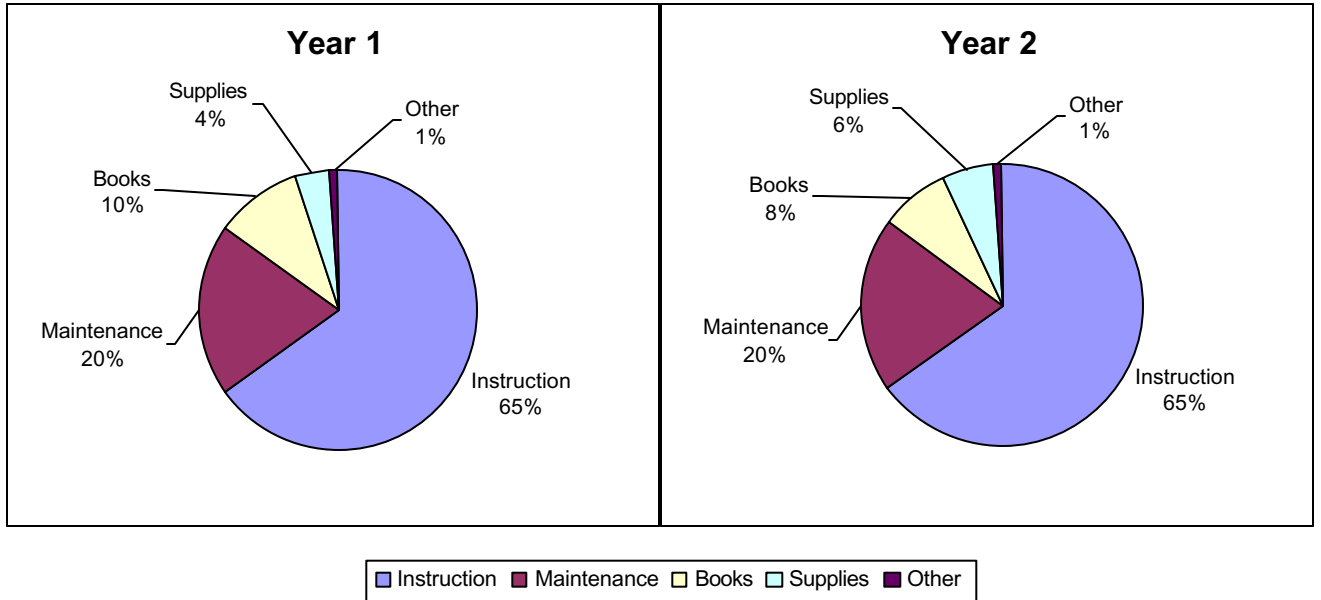
Package 1	Package 2	Package 3	Package 4
Dues: \$6.99 Each CD: \$11.50	Dues: \$5.99 Each CD: \$12.99	Dues: \$2.99 Each CD: \$13.50	Dues: \$4.99 Each CD: \$13.20

- A** Package 1
B Package 2
C Package 3
D Package 4

High School—Part I

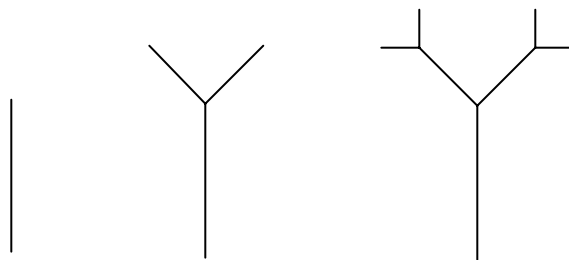
- 3 The circle graphs represent school district spending for 2 successive years. The school budget for the first year was \$30,500,000 and \$32,500,000 for the second. How much more money was spent on supplies in Year 2 than in Year 1?

School Budget



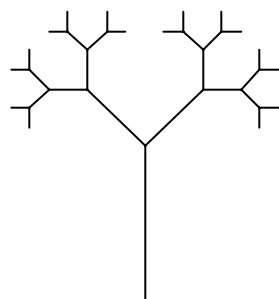
- A** \$610,000
B \$650,000
C \$730,000
D \$1,260,000

- 4 Jason created a pattern of “trees” by adding two segments to each branch. Study the first three elements of his pattern.

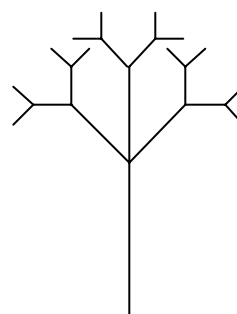


Which of the following will be an element of the pattern?

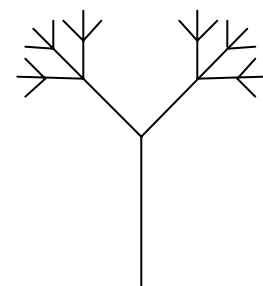
A



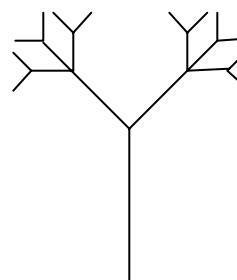
B



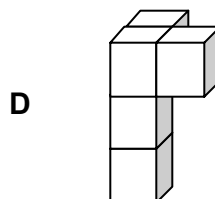
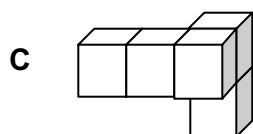
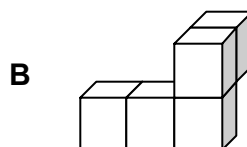
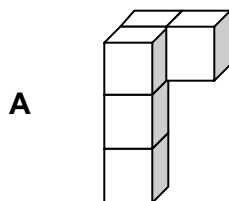
C



D



- 5 Which pentacube is **not** congruent to the others?



- 6 Colonel Crisp contains one of 6 different prizes in each box of cereal. The prizes are distributed evenly and randomly. If two boxes of the cereal are purchased, what is the probability that the prizes will be different?

A $\frac{2}{3}$

B $\frac{5}{6}$

C $\frac{6}{5}$

D $\frac{1}{6}$

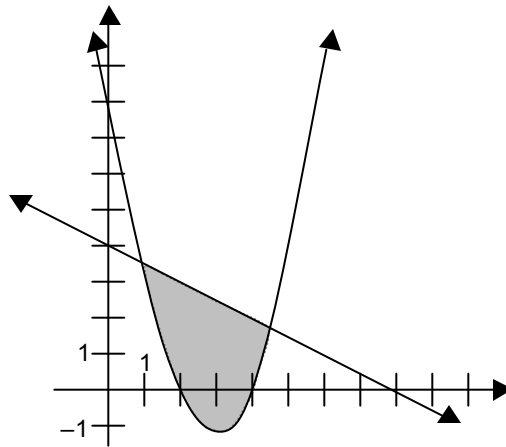
High School—Part I

- 7** The distance required to stop a moving vehicle after applying the brakes varies directly as the square of the vehicle's speed. If a vehicle traveling at 20 mph requires 50 feet to stop, what is the stopping distance for a vehicle traveling at 60 mph?
- A** 150 ft
 - B** 300 ft
 - C** 450 ft
 - D** 480 ft
- 8** Thirty-two competitors begin a chess tournament. Trophies are awarded to first, second, and third places. How many different sets of first, second and third place winners are possible?
- A** 93
 - B** 1893
 - C** 29,760
 - D** 32,768

High School—Part I

- 9** Pete currently earns \$500 per month. He receives a 4% salary increase every 6 months. Three years from now, which amount will be closest in value to Pete's monthly salary?
- A** \$560
 - B** \$630
 - C** \$800
 - D** \$1000
- 10** To compute the length of the hypotenuse of an isosceles right triangle, multiply the length of a leg by the square root of 2. If the length of one leg is a prime number, what type of number is the length of the hypotenuse?
- A** composite
 - B** irrational
 - C** prime
 - D** whole

- 11 Which two inequalities does this graph represent?



- A** $y \geq (x + 3)^2 + 1$ and $y \leq -\frac{1}{2}x + 4$
- B** $y \leq (x - 3)^2 + 1$ and $y \leq \frac{1}{2}x + 4$
- C** $y \leq (x + 3)^2 - 1$ and $y \geq -2x + 4$
- D** $y \geq (x - 3)^2 - 1$ and $y \leq -\frac{1}{2}x + 4$

Team Record

Game	1	2	3	4	5
Points Scored	57	34	61	58	45

- 12 The team statistician calculated the standard deviation from the mean to be approximately 10 points. In game 6, the team scored 56 points. How did this score compare to the mean of the first five games?
- A** about one standard deviation above the mean
- B** about one-half standard deviation below the mean
- C** about one-half standard deviation above the mean
- D** about one standard deviation below the mean

High School—Part I

- 13** The set $x = \{ 2, 5, 1, 9 \}$ represents the number of gallons of fuel used each day by a machine during the past four days. The number of gallons of fuel it uses on a given day determines the number of items the machine produces that day. The number of items produced each day is modeled by the following formula:

$$I(x) = x^2 - 14x + 45$$

Which value of x , used in the last four days, produced the most items?

- A** 1
B 2
C 5
D 9
- 14** The chart shows the enrollment of students in certain classes at Jefferson High School. Assume no student is enrolled in more than one course per subject area.

Jefferson High School Enrollment

	Mathematics	Science	Social Studies	English
Freshmen	458	471	352	685
Sophomores	402	507	279	701
Juniors	378	398	568	699
Seniors	125	366	221	712

Based on the information in the chart, which conclusion is correct?

- A** There are 1966 freshmen at Jefferson High School.
B All seniors take English at Jefferson High School.
C The junior class has the greatest number of students at Jefferson High School.
D There are 1742 students enrolled in science courses at Jefferson High School.

- 15** Students in a shop class are building a step pyramid using blocks that measure 4 inches \times 4 inches \times 4 inches. The base is a border of 20 blocks as shown in Figure 1 in the diagram.

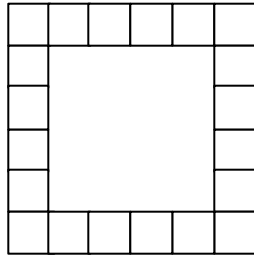


Figure 1

Each successive level contains blocks that overlap by 2 inches as shown in Figure 2 in the diagram.

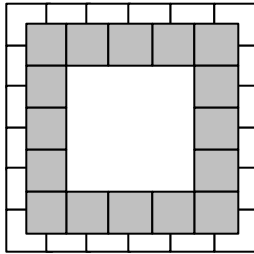


Figure 2

- A** Describe the pattern of the number of blocks as the pyramid is built from one level to the next.
- B** If the pattern continues, how many levels of blocks will the completed pyramid have?
- C** What is the total number of blocks in the completed pyramid?

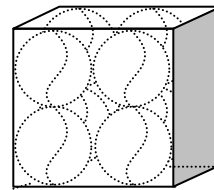
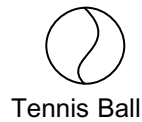
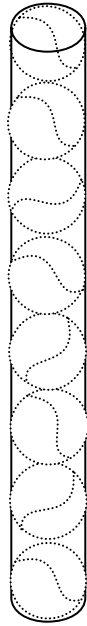
Explain your answers, including supporting calculations, tables, diagrams, charts, drawings/graphs.

High School—Part I

- 16** Keith builds two types of chairs for his furniture business. A folding chair sells for \$50 and a dining chair sells for \$120.
- A** Last month Keith sold a total of 30 chairs and earned \$3180. How many of each type of chair did he sell?
- B** Next month Keith plans to sell $\frac{1}{3}$ more dining chairs and $\frac{1}{3}$ fewer folding chairs. How much will he earn according to that plan?

Explain your answers, including supporting calculations, tables, diagrams, charts, drawings/graphs.

- 17** The design team of the Tennis Ball Company is considering two different containers, one cylindrical and one cubical. Each container must hold 8 balls that measure 2 inches in diameter. (See picture below.)



- A** Which container requires less material to manufacture?
- B** What is the difference in value of the surface areas of the material used for each container?

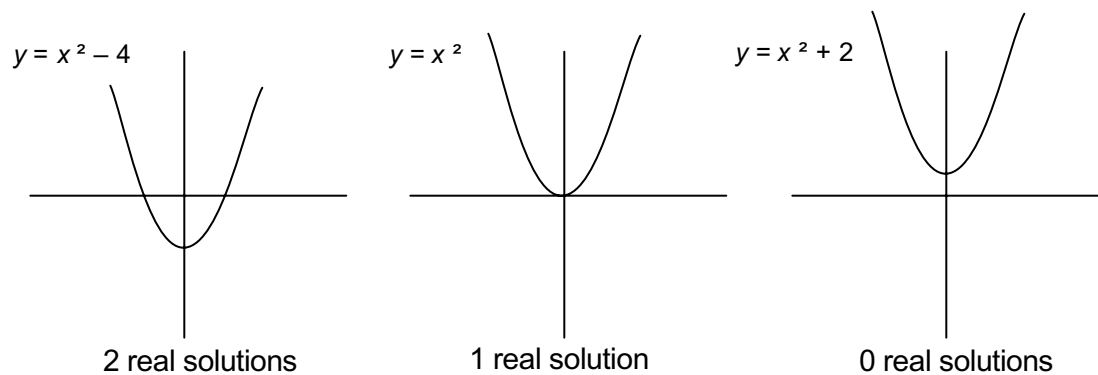
Explain your answers including supporting calculations, tables, diagrams, charts, drawings/graphs.

- 18** Melissa is organizing the next school dance. She plans to survey the students at her school regarding their music preferences. Which approach for surveying students would be least biased?
- A** Survey all athletes.
 - B** Survey her friends and teachers.
 - C** Survey the other people on the dance committee.
 - D** Survey a random selection of 50 students.
- 19** List the following real numbers in order from greatest to least.

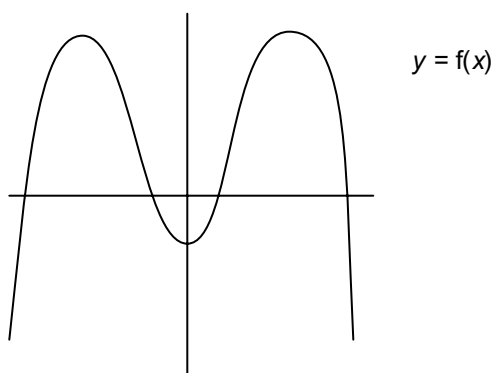
$$3\pi \qquad \sqrt{69} \qquad \frac{33}{8} \qquad 8.14$$

- A** $3\pi, \sqrt{69}, 8.14, \frac{33}{8}$
- B** $3\pi, 8.14, \sqrt{69}, \frac{33}{8}$
- C** $3\pi, \sqrt{69}, \frac{33}{8}, 8.14$
- D** $\sqrt{69}, \frac{33}{8}, 8.14, 3\pi$

- 20** Look at these graphs.

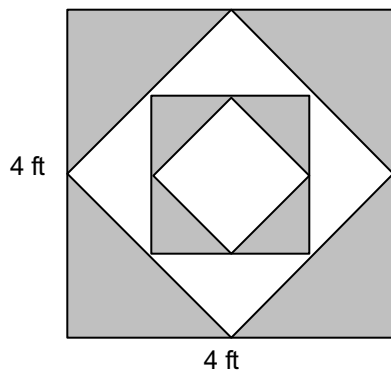


How many real solutions exist for the following graph?



- A** 2
- B** 3
- C** 4
- D** 5

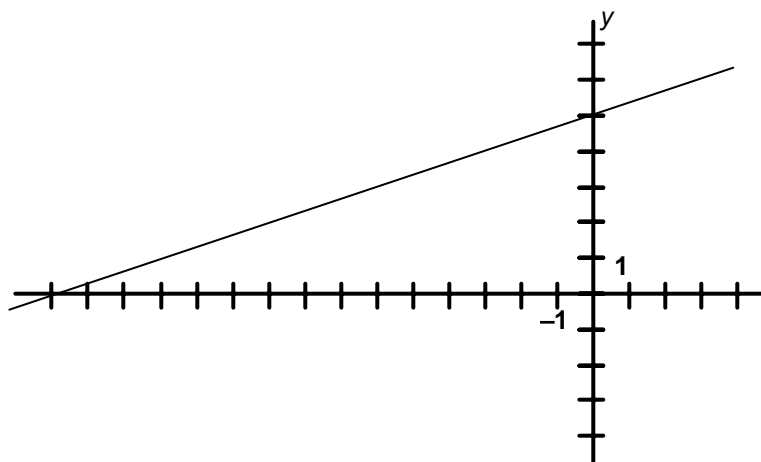
- 21** Matt is creating a game for a charity event. The game board is shown. The midpoints of the sides of the outer square are the vertices of the inner squares.



Find the total area of the shaded regions.

- A** 6 square feet
- B** 8 square feet
- C** 10 square feet
- D** 12 square feet

- 22 What is the rate of change of the line shown on the graph?



- A $\frac{1}{5}$
 B $\frac{1}{3}$
 C 3
 D 5
- 23 A rectangle is cut into four pieces as shown. The areas of the three pieces are indicated.

	y	z
a	6	10
b	9	

What is the area of the original rectangle?

- A 13
 B 35
 C 40
 D 540

- 24** The table shows the speed of sound in air at different temperatures.

Speed of Sound at Different Temperatures

Air Temperature	Speed of Sound
16°C	1118 ft/sec
20°C	1126 ft/sec
24°C	1134 ft/sec

How far will sound travel in 20 seconds in air that is 29°C?

- A** 22,780 ft
B 22,840 ft
C 22,860 ft
D 22,880 ft
- 25** When two regular octahedrons (8-faced polyhedra) with faces numbered 1–8 are rolled, which sum is most likely?
- A** 8
B 9
C 12
D 16

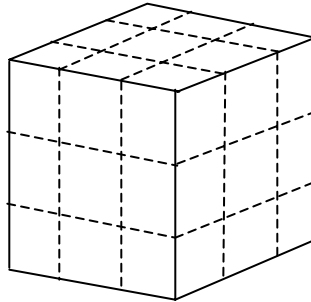
- 26** The following functions represent the population of field mice at the beginning of a study in Regions A and B, where t represents the number of years from the beginning of the study.

$$A(t) = 3t^2 + 10t + 15$$

$$B(t) = 3t^2 + 2t + 10$$

After how many years is the difference in population between Region A and Region B equal to 29?

- A** $\frac{1}{3}$
B $\frac{1}{2}$
C 2
D 3
- 27** The figure represents a wooden block 3 inches on each edge, all of whose faces are red. If the block is cut along the dotted lines, 27 blocks result, each measuring one cubic inch in volume.



Of these, how many will have only two red faces?

- A** 8
B 12
C 16
D 24

- 28** How many 4-letter permutations of LIFE are possible?

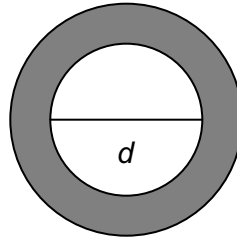
A 12
B 16
C 20
D 24

- 29** For the data table, determine the rule when $x = n$.

x	1	2	3	4	5
y	25	20	15	10	5

- A** $25 + 5n$
B $25 - 5n$
C $30 + 5n$
D $30 - 5n$
- 30** Jenny saved \$100 to invest. If she deposits the money into a savings account that yields 6% per year, how much money will be in the account in 5 years? Interest is compounded annually.
- A** \$130.00
B \$133.82
C \$134.01
D \$136.00

- 31** Bob used x sheets of a 500-sheet pack of typing paper. Mike used twice as much as Bob. Pete used half as much as Mike. Which algebraic expression represents the amount of paper remaining?
- A** $4x$
B $3\frac{1}{2}x$
C $500 - 4x$
D $500 - 3\frac{1}{2}x$
- 32** A hardware company manufactures circular washers. The company has a precise standard for the 24 mm hole in their washers. They use the rule $|d - 24| \leq 0.05$, where d is the diameter (in millimeters) of the inside hole.



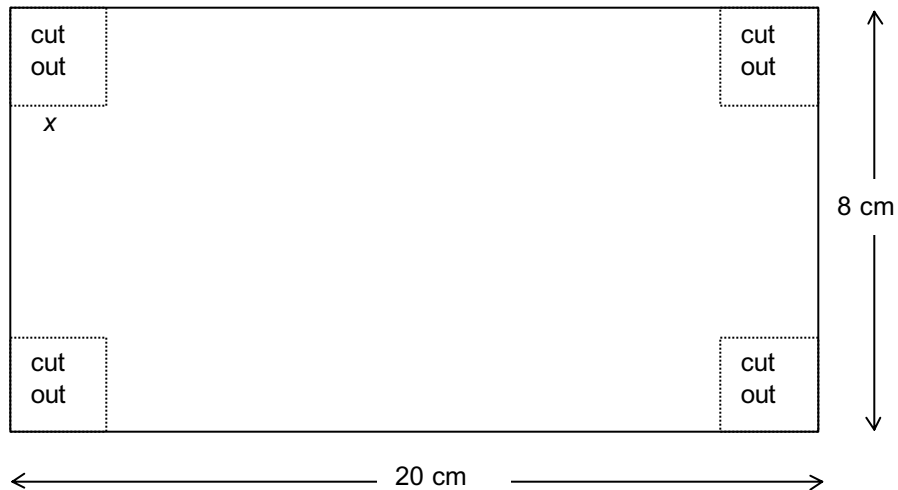
Circular Washer

Which of these diameters would be **unacceptable** to the manufacturer?

- A** 23.60 mm
B 23.95 mm
C 23.96 mm
D 24.03 mm

High School—Part II

- 33** The number of teeth in the gears of a clock is inversely proportional to the turning speed. Gear 1 has 60 teeth and turns at 450 rpm. Gear 2 has 36 teeth. What is the approximate turning speed of gear 2?
- A** 270 rpm
B 484 rpm
C 750 rpm
D 864 rpm
- 34** A manufacturer wishes to make an open box from a rectangular piece of cardboard. She will cut square corners out of the piece as indicated in the picture.



The length of the side of one of these cut-out squares is x . The cardboard is to be folded up at the corners to create the box. Which of the following gives a model for the volume of the box?

- A** $x(20 - 2x)(8 - 2x)$
B $x(10 - x)(4 - x)$
C $x(x - 8)(x - 20)$
D $160x$

- 35** Consider the following stem-and-leaf plot. How many observations are in the data?

Observations	
5	1
4	4 5 7
3	0 0 3 6
2	1 1 3 4 5 9 9
1	2 2 4 8
0	0 1 2

- A** 5
B 7
C 15
D 22
- 36** Three teenagers (T1, T2, T3), two children (C1, C2), and an adult (A) are in line at an amusement park. Only two individuals are selected each time for the bumper car ride.

A List all possible outcomes for the selection.

B What is the probability that a teenager and a child will be selected for a given ride?

Explain your answers, including all supporting calculations, tables, diagrams, charts, drawings/graphs.

- 37** Mark took a typing test 10 times. The table shows his average speed and error rate for each test.

TYPING TEST RESULTS

Test	Average Speed (words per minute)	Error Rate (errors per page)
1	36	3
2	41	4
3	52	16
4	48	13
5	43	4
6	39	2
7	47	15
8	40	6
9	51	11
10	46	12

- A** Create a graph showing the relationship between typing speed and error rate for these data.

- B** Describe Mark's error rate as he increased his average typing speed.

Explain your answers, including supporting calculations, tables, diagrams, charts, drawings/graphs.

PART THREE: SCORING

This section includes sample student responses, scoring guides, and annotations on scoring for each score point on each open-ended item.

It is important to note that these sample student responses are not the only way a student can earn a particular score. They are representative of the types of responses elicited by the item. The focus is on how to relate a specific response to the scoring guide for that problem.



#15

STUDENT RESPONSES AND SCORING

A **4-point** response includes all of the following components:

- Demonstrates understanding of the structure of the pattern.
- Determines there are 6 levels of blocks required to complete the pyramid, with a total of 61 blocks. An answer of 5 levels and 60 blocks is also acceptable. (i.e., the one block on top is unnecessary).
- Provides a clear, valid description of the pattern.
- Clearly provides all supporting work and explanations.

A **3-point** response includes the following:

- Determines there are 6 levels of blocks required to complete the pyramid, with a total of 61 blocks. An answer of 5 levels and 60 blocks is also acceptable (i.e., the one block on top is unnecessary).
- Provides a clear, valid description of the pattern.
- Demonstrates understanding of the pattern, but makes some minor errors (such as miscounting or calculation errors) when determining the number of levels and/or the number of blocks.

OR

- Provides a description of the pattern that is vague or only partially correct.
- If the student fills in the pyramid with blocks that is considered a minor error and the response may score as high as a “3,” as long as they clearly indicate they are filling in the pyramid.

A **2-point** response includes the following or similar:

- Demonstrates partial understanding of the pattern, but makes errors when determining the number of levels and/or the number of blocks.

AND

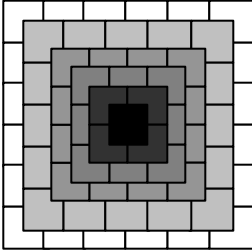
- Provides a description of the pattern that is vague or only partially correct.

A **1-point** response includes the following or similar:

- Demonstrates limited understanding of the pattern.
- Attempts to determine the number of levels and/or blocks, but the answers are significantly flawed.
- The description of the pattern is incorrect or missing.

A **0-point** response shows little or no understanding of the task.

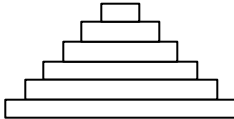
Student Response (4 points)



6	5	4	3	2	1	level (blocks on a side)
20	16	12	8	4	1	# blocks

A. Each row/layer has one fewer block on each side than the row before it. The bottom has 6 blocks on a side, the second level has 5 block, etc.

B. There will be a total of six levels of blocks to finish the pyramid. After that it would be a pyramid with a tower on top if you kept building, like so:



finished

C. Sixty-one (61) blocks will be in the finished pyramid; 20 at the bottom, then 16, then 12, then 8, then 4, then finally, 1 block on the very top.

A

B

C

Score 4

This is a 4-point response because

- (A) The student demonstrates understanding of the structure of the pattern by recognizing that each layer has 1 less block than the layer below it.
- (B) The student recognizes that there are 6 levels of blocks.
- (C) The student correctly determines that there are 61 total blocks in the pyramid.

Each response, **A**, **B**, and **C** provides a clear, valid description of the pattern and all supporting work and calculations are shown.

Student Response (3 points)

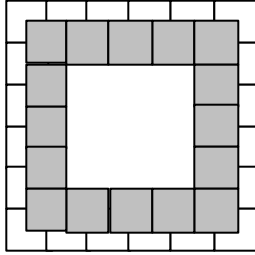
<p>A. They overlap about half the block under it and soon they form a point. The pattern is shown in the numbers in Part B.</p>	A
<p>B. Level 1 20 6 per side Level 2 16 5 per side Level 3 12 4 per side Level 4 8 3 per side Level 5 4 2 per side Level 6 1 1</p>	B
<p>C. $20 + 16 + 12 + 8 + 4 + 1 = 71$</p>	C

Score 3

This is a 3-point response because

- (A) The student demonstrates understanding of the pattern, and explains the relationship between the number of blocks in each level and the number in the succeeding level in part B.
- (B) The student shows 6 levels of blocks.
- (C) Response has a minor computation error. Does not match the explanation of the pattern.

Student Response (2 points)

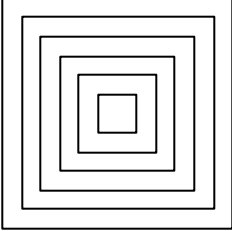
<div data-bbox="643 321 896 573"></div> <p data-bbox="282 604 1170 684">A. It starts with having 6 by 6, then 5 by 5, then 4 by 4, then 3 by 3, then 2 by 2, and finally 1 by 1. This makes a pyramid.</p> <p data-bbox="282 730 448 764">B. Six levels</p> <p data-bbox="282 856 440 890">C. 91 blocks</p>	<p data-bbox="1317 617 1338 646">A</p> <p data-bbox="1317 726 1338 756">B</p> <p data-bbox="1317 835 1338 865">C</p>
--	---

Score 2

This is a 2-point response because

- (A) The description of the pattern is vague.
- (C) Demonstrates some understanding of the pattern, but makes an error in determining the number of blocks.

Student Response (1 point)

<p><i>A.</i></p>  <p><i>B. 5</i></p> <p><i>C. 61</i></p>	<p>A</p> <p>B</p> <p>C</p>
---	---

Score 1

- (A) The drawing represents a limited understanding of the pattern.
- (B) The number of levels and/or blocks is incorrect and contains no supporting work to indicate the source of the answer.
- (C) The total number of blocks is correct.

Student Response (0 points)

<i>A. There is 3 less blocks on each level and the pyramid is cut in half with each layer.</i>	A
<i>B. 5 layers</i>	B
<i>C. $20 + 17 + 14 + 11 + 8 = 70$ blocks</i>	C

Score 0

This is a 0-point response because

- (A) Incorrect relationship of number of blocks to level indicates little or no understanding.
- (B) Indicates no understanding of pattern.
- (C) Indicates no understanding of relationship of number of blocks in each layer to total.

#16

STUDENT RESPONSES AND SCORING

A **4-point** response includes all of the following components:

- For Part A, applies a valid strategy to determine 6 folding chairs and 24 dining chairs were sold. An example of a system of equations that could be used is $F + D = 30$, $50F + 120D = 3180$.
- For Part B, applies a valid strategy to determine that selling one-third more dining chairs (32 dining chairs) and one-third fewer folding chairs (4 folding chairs) would earn \$4040.
- Clearly provides all supporting work and calculations.

A **3-point** response meets most of the criteria, but may do one of the following or similar:

- Attempts to apply valid strategies for solving the problems, but makes 1 or 2 minor errors that result in incorrect answers. Clearly provides all supporting work and calculations.
- Provides correct answers for Parts A & B, but some of the work is incomplete.

A **2-point** response includes the following or similar:

- Attempts to apply valid strategies to solve the problem, but makes significant errors that result in incorrect answers.
- Provides some supporting work and calculations.

A **1-point** response includes one of the following or similar:

- Attempts to apply a strategy to solve the problem, but the strategy is incomplete or unclear.
- Provides one or two correct answers, but shows no work.

A **0-point** response shows little or no understanding of the task.

*A response receives one point for giving the correct formula ($50F + 120D = 3180$) and attempts to solve for a variable. However just giving the formula with no attempt to solve for a variable does not receive credit.

Student Response (4 points)

<p>A. $50(x) + 120(y) = 3180$</p> <p>$x + y = 30$</p> <p>$50(x) + 120(y) = 3180$</p> <p>- $120(y)$</p> <p>- $50(x)/50 = 3180/50 - 120(y)/50$</p> <p>- $x = 63.6 - 2.4y$</p> <p>- $((63.6 - 2.4y) + y) = 30$</p> <p>- $-63.6 \quad -63.6$</p> <p>- $-2.4y + y = -33.6$</p> <p>- $-1.4y = -33.6$</p> <p>$1.4y/1.4 = 33.6/1.4$</p> <p>$y = 24$</p> <p>$24(120) = 2880$</p> <p>$3180 - 2880 = 300/50 = 6$</p> <p>6 Folding chairs were sold</p> <p>24 Dining chairs were sold</p> <p>B. $24(1/3) = 8$</p> <p>$8 + 24 = 32(120) = 3840$</p> <p>$6(1/3) = 2$</p> <p>$6 - 2 = 4(50) = 200$</p> <p>$3840 + 200 = 4040$</p> <p>He will earn \$4040.</p>	<p>A</p> <p>B</p>
---	---------------------------------

Score 4

This is a 4-point response because

- (A) The student has applied a valid strategy to determine 6 folding chairs and 24 dining chairs were sold. The system of equations is of the type $F + D = 30$ and $50F + 120D = 3180$.
- (B) The student has applied a valid strategy to find that selling one-third more dining chairs and one-third fewer folding chairs would earn \$4040.

Student Response (3 points)

<p>A. $24 \times 120 = 2880$</p> <p>$6 \times 50 = \frac{300}{3180}$</p> <p>B. $24 \times (1/3) = 8$</p> <p>$24 + 8 = 32 \times 120 = 3840 + 400 = 4240$</p> <p>$6 \times (1/3) = 2$</p> <p>$6 + 2 = 8$</p> <p>$8 \times 50 = 400$</p> <p><i>24 dining chairs</i> <i>6 folding chairs</i></p> <p><i>He will earn \$4240</i></p>	<p>A</p> <p>B</p>
--	---------------------------------

Score 3

This is a 3-point response because

- (A) The student applied a valid strategy, but made a minor error that resulted in an incorrect answer.
- (B) The calculated earnings are based on the error.

Student Response (2 points)

$A. 50f + 120d = 3180$ $f + d = 30$ $f = 6 \quad d = 24$	A
$B. 1/3 \times 6 = 2 \quad 6 + 2 = 8 \quad (50 \times 8) + (120 \times 16) = \2320.00 $1/3 \times 24 = 8 \quad 24 - 8 = 16$	B

Score 2

This is a 2-point response because

- (A) The student attempts a valid strategy. The student does not provide supporting work.
(B) The student makes significant errors that result in incorrect answers.

Student Response (1 point)

<p>A. $\mathcal{Q} = 50 \quad \mathcal{D} = 120$</p> <p>$\mathcal{Q} + \mathcal{D} = 30$</p> <p>$50\mathcal{Q} + 120\mathcal{D} = 3180$</p> <p>$\mathcal{D} = 265 - 50\mathcal{Q}$</p> <p>$\mathcal{D} = 30 - \mathcal{Q}$</p> <p>$30 - \mathcal{Q} = 265 - 50\mathcal{Q}$</p> <p>$50\mathcal{Q} = 235$</p> <p>4.7 folding chairs</p> <p>25.3 dining chairs</p>	A
<p>B. \$4190.43</p>	B

Score 1

This is a 1-point response because

- (A) The student attempts to solve the problem, but the strategy is incomplete. The student has developed an appropriate equation, but failed to recognize that non integer results are inappropriate. The total is calculated incorrectly.
- (B) No supporting work is provided.

Student Response (0 points)

<p><i>Folding chair</i> \$50</p> $\begin{array}{r} 50 \\ \times \cancel{20} \\ \hline 00 \\ + 1000 \\ \hline 1000 \end{array}$ $\begin{array}{r} 50 \\ \times \cancel{40} \\ \hline 00 \\ + 2000 \\ \hline 2000 \end{array}$ $\begin{array}{r} 50 \\ \times \cancel{42} \\ \hline 100 \\ + 2000 \\ \hline 2100 \end{array}$	<p><i>Dining chair</i> \$120</p> $\begin{array}{r} 50 \\ \times 5 \\ \hline 250 \\ \times 5 \\ \hline 1250 \end{array}$ $\begin{array}{r} 600 \\ + 250 \\ \hline 850 \\ + 1250 \\ \hline 2100 \end{array}$	<p>= \$3180</p> $\begin{array}{r} 120 \\ \times 5 \\ \hline 600 \end{array}$ $\begin{array}{r} 120 \\ \times 8 \\ \hline 960 \end{array}$ $\begin{array}{r} 120 \\ \times 4 \\ \hline 480 \end{array}$ $\begin{array}{r} 120 \\ \times 10 \\ \hline 000 \\ 1200 \end{array}$ $\begin{array}{r} 120 \\ \times 9 \\ \hline 1080 \end{array}$ $\begin{array}{r} 3180 \\ - 2100 \\ \hline 1080 \end{array}$ $\begin{array}{r} 3180 \\ - 1080 \\ \hline 2100 \end{array}$
<p>Answer 9 dining chairs and 4.2 folding chairs</p>		

Score 0

This is a 0-point response because
Student attempts to guess by multiplication; shows little understanding of the task.

#17

STUDENT RESPONSES AND SCORING

A **4-point** response includes all of the following components:

- Demonstrates understanding of both surface areas.
- Correctly calculates the surface area of each container, with the cylinder having a surface area of 106.76 sq in and the cube having a surface area of 96 sq in.
[Note: The answers may be slightly different, depending on whether or not student rounds answers throughout the calculations.]
- Indicates the cube requires less material than the cylinder.
- Calculates the difference in the surface areas of the material used is 10.76 sq in.
- Clearly provides all supporting work and calculations.

A **3-point** response includes the following or similar:

- Demonstrates understanding of both surface areas.
- Applies a valid strategy to solve both parts of the problem, but either makes 1 or 2 minor errors in the calculations or shows little or no work.
- Correctly indicates which container requires less material, depending on calculations OR correctly calculates the difference of surfaces based on calculations..
- Provides sufficient supporting work and calculations.

A **2-point** response includes the following or similar:

- Demonstrates some understanding of both surface areas.
- Attempts to apply a valid strategy to solve the problem, but makes significant errors.
- Provides some supporting work and calculations.

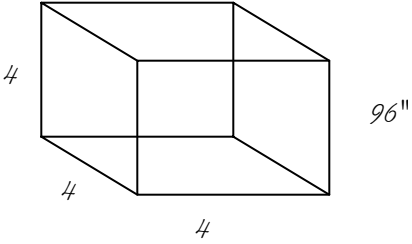

A **1-point** response includes the following or similar:

- Demonstrates limited understanding of both surface areas.
- Attempts to apply a strategy to solve the problem, but the strategy is incomplete, unclear, or contains major flaws.

A **0-point** response shows little or no understanding of the task.

- May only understand one surface area.

Student Response (4 points)

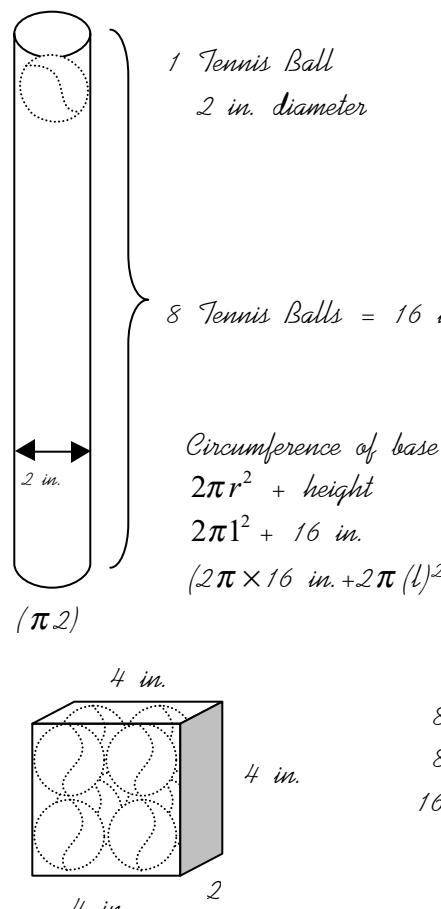
	$\begin{array}{r} 16 \\ \times 6 \\ \hline 96 \end{array}$		A
<p>A) One cubical has less material</p> <p>B) 106.814</p> $\begin{array}{r} 106.814 \\ - 96.000 \\ \hline 10.814 \text{ inches}^2 \end{array}$	$\begin{aligned} c &= \pi D \\ &= 2\pi \\ 2\pi r^2 &= 2\pi l^2 \\ &= 2\pi \end{aligned}$	$\begin{array}{r} 16'' \\ \times 2\pi \\ \hline 32\pi + 2\pi \\ = 34\pi \\ \approx 106.814 \end{array}$	B

Score 4

This is a 4-point response because

- (A) The student has correctly calculated the surface area of each container; cylinder = 96 square inches, cube ≈ 106.8 square inches. This response also indicates that the cubical shape uses less material.
- (B) The response also gives the difference between the two surface areas as approximately 10.8 square inches.

Student Response (3 points)

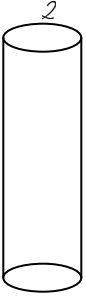
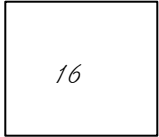
 <p>1 Tennis Ball 2 in. diameter</p> <p>8 Tennis Balls = 16 in.</p> <p>Circumference of base $2\pi r^2 + \text{height}$ $2\pi 1^2 + 16 \text{ in.}$ $(2\pi \times 16 \text{ in.} + 2\pi (1)^2) = 106.8 \text{ in.}^2$</p> <p>($\pi 2$)</p> <p>4 in.</p> <p>4 in.</p> <p>4 in.</p> <p>2</p> <p>8 × 2 = 16 8 × 2 = 16 16 × 2 = + 32 64 in.²</p> <p>A. The prism requires less material.</p> <p>B. The difference in the surface areas is approximately 43 in.²,</p>	<p>A</p> <p>B</p>
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Score 3

This is a 3-point response because

- (A) This response indicates understanding of surface area. It contains a valid strategy to solve both parts of the problem. The student has correctly calculated the surface area of the cylinder as 106.8 square inches. However in attempting to calculate the surface area of the cube, the student has mistakenly used 2 inches as one of the dimensions of the cube. This has resulted in an incorrect surface area for the cube, 64 square inches. The response also indicates that the prism (cube) requires less material.
- (B) The incorrect calculation of the difference in surface areas is based on the 2-inch dimension in the drawing.

Student Response (2 point)

<p>A. The cube has less surface area, therefore it requires less material.</p> <p>B. The cube has a SA of 96 in² and the cylinder has a SA of 100.48 in²</p> <div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <p>Circumference = $2\pi r = 6.28$</p> <p>$6.28 \times 16 = 100.48$ S.A.</p> </div> </div> <div style="display: flex; align-items: center; margin-top: 20px;">  <div style="margin-left: 20px;"> <p>$16 \times 6 = 96$ S.A.</p> </div> </div>	<p>A</p> <p>B</p>
--	---------------------------------

Score 2

This is a 2-point response because

- (A) The student has demonstrated some understanding of surface area, but has made a significant error. In calculating the surface area of the cylinder, the student forgot to add the area of the bases. The surface area of the cube is correct (96). The student has shown some supporting work.
- (B) The student does not attempt to state the difference between the surface areas.

Student Response (1 points)

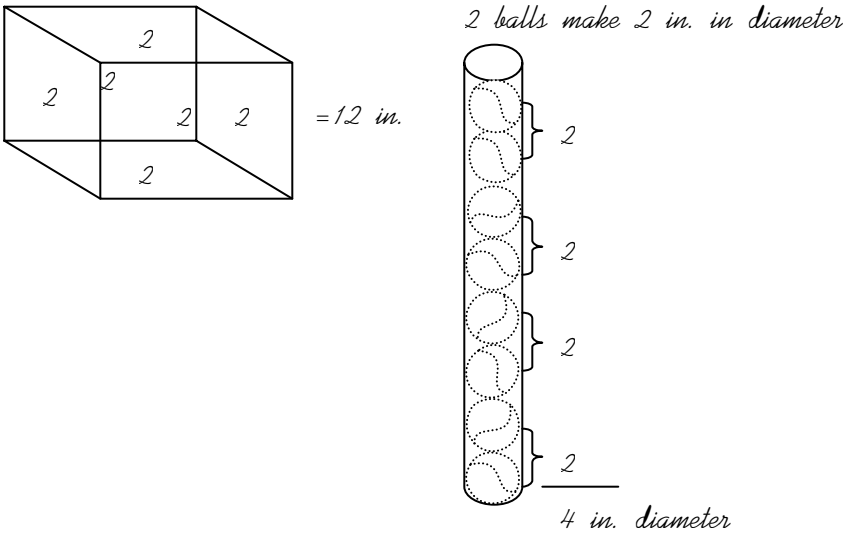
<p>A.</p> $V \text{ (cylinder)} = \pi r^2 \times h$ $= \pi 2^2 \times 32$ $V = 402.12 \text{ inches}^3$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">the cube.</div>	<p>A</p>
<p>B.</p> $SA \text{ (cylinder)} = 2\pi r \times h + 2\pi r^2$ $= 2\pi 2 \times 32 + 2\pi 2^2$ $= 441$ $- 96$ <hr style="width: 50%; margin-left: 0;"/> <div style="border: 1px solid black; padding: 2px; display: inline-block;">345</div>	<p>B</p>

Score 1

This is a 1-point response because

- (A) The student has demonstrated limited understanding of surface area by trying to compute the volume to determine which container uses less material.
- (B) The student correctly determines the surface area of the cube, but makes a major flaw in computing the surface area of the cylinder. The student uses the wrong formula and radius.

Student Response (0 points)

<p>A. The cylindrical</p>  <p>B. The box has 6 sides or surface areas, the tube only has 1, so less material.</p>	<p>A</p> <p>B</p>
---	---------------------------------

Score 0

This is a 0-point response because

- (A) This response indicates no understanding of surface area. There is no relationship between 12 inches and the surface area of the cube.
- (B) The 4-inch diameter indicated for the cylinder is incorrect and not related to its surface area.

#36

STUDENT RESPONSES AND SCORING

A **4-point** response does the following:

Part A is complete and Part B is correct based on the information in Part A.

A **3-point** response does the following:

Part A is complete and Part B is partial.

OR

Part A is partial and Part B is correct based on the information in Part A.

A **2-point** response does the following:

Both Part A and Part B are partial.

OR

Part A is complete and Part B is either missing or incorrect.

OR

Part A contains too many errors to receive partial credit, but Part B is correct based on the information in Part A.

A **1-point** response does the following:

Part A is partial and Part B is missing or incorrect.

OR

Part A contains too many errors to receive partial credit, but Part B is partial.

A **0-point** response shows little or no understanding of the task.

Part A:

Complete = all 15 pairings (T1, T2), (T1, T3), (T1, C1), (T1, C2), (T1, A), (T2, T3), (T2, C1), (T2, C2), (T2, A), (T3, C1), (T3, C2), (T3, A), (C1, C2), (C1, A), (C2, A)

[Note: The student may use different notation to represent the elements of the set, but the notations must be clear.]

Partial = 12 to 14 pairings

No credit = 11 or less

Part B:

Complete = correct answer ($6/15 = 2/5 = 0.4$)

Partial = either the 6 or the 15 is incorrect (based on A)

OR

The probability is a correct value but is not given in the correct form.

Student Response (4 points)

<p>A.</p> <table> <tr> <td>$T_1 T_2$</td> <td>$T_2 T_3$</td> <td>$T_3 C_1$</td> <td>$C_1 C_2$</td> </tr> <tr> <td>$T_1 T_3$</td> <td>$T_2 C_1$</td> <td>$T_3 C_2$</td> <td>$C_1 A$</td> </tr> <tr> <td>$T_1 C_1$</td> <td>$T_2 C_2$</td> <td>$T_3 A$</td> <td>$C_2 A$</td> </tr> <tr> <td>$T_1 C_2$</td> <td>$T_2 A$</td> <td></td> <td></td> </tr> <tr> <td>$T_1 A$</td> <td></td> <td></td> <td></td> </tr> </table> <p>each person can be paired with any other person</p> $\frac{6}{15} = \frac{2}{5}$	$T_1 T_2$	$T_2 T_3$	$T_3 C_1$	$C_1 C_2$	$T_1 T_3$	$T_2 C_1$	$T_3 C_2$	$C_1 A$	$T_1 C_1$	$T_2 C_2$	$T_3 A$	$C_2 A$	$T_1 C_2$	$T_2 A$			$T_1 A$				<p>A</p>
$T_1 T_2$	$T_2 T_3$	$T_3 C_1$	$C_1 C_2$																		
$T_1 T_3$	$T_2 C_1$	$T_3 C_2$	$C_1 A$																		
$T_1 C_1$	$T_2 C_2$	$T_3 A$	$C_2 A$																		
$T_1 C_2$	$T_2 A$																				
$T_1 A$																					
<p>B. 6 of the possible combinations have a child and a teenager in them.</p>	<p>B</p>																				

Score 4

This is a 4-point response because

(A) Part A is complete.

(B) Part B is correct based on the information in Part A.

Student Response (3 points)

<p>3T, 2C, 1A, T = teenager C = children A = Adult</p> <p>A. T1T2, T1T3, T2T3, T1C1, T1C2, T2C1, T2C2, T3C1, T3C2, T1A, T2A, T3A, C1A, C2A = 14 ways</p> <p>B. $P = 3/7$ } probability that a teenager and a child is selected (6/14)</p>	<p>A</p> <p>B</p>
---	---------------------------------

Score 3

This is a 3-point response because

- (A) Part A is partial—14 correct pairings.
- (B) Part B is correct based on the information in Part A.

Student Response (2 points)

<u>T₁ T₂ T₃ C A</u>	A
A.	
T ₁ T ₂	
T ₁ T ₃	
T ₁ C	
T ₁ A	
T ₂ T ₃	
T ₂ C	
T ₂ A	
T ₃ C	
T ₃ A	B
C A	
3/10 = 30 % chance	

Score 2

This is a 2-point response because

- (A) Part A has too few pairings to receive credit.
(B) Part B is correct based on the information in Part A.

Student Response (1 point)

<i>A. Teenager-child Teenager-teenager child-child Teenager-adult Child-Adult</i>	A
<i>B. 1 out of 6 or 17%</i>	B

Score 1

This is a 1-point response because

(A) Part A has too few pairings to receive partial credit.

(B) Part B is partial based on Part A. The 1 is correct, but the 6 is not.

Student Response (0 points)

$3t$ $2c$ A	
$2t$	
$1t$ & $1c$	
$1t$ & $1a2c$	
$2c$ & $1a$	
? 83% $5/6$	

Score 0

This is a 0-point response because

So few outcomes are listed in the sample space, the student indicates little understanding of the task. The probability calculation is based on no part of the student's work.

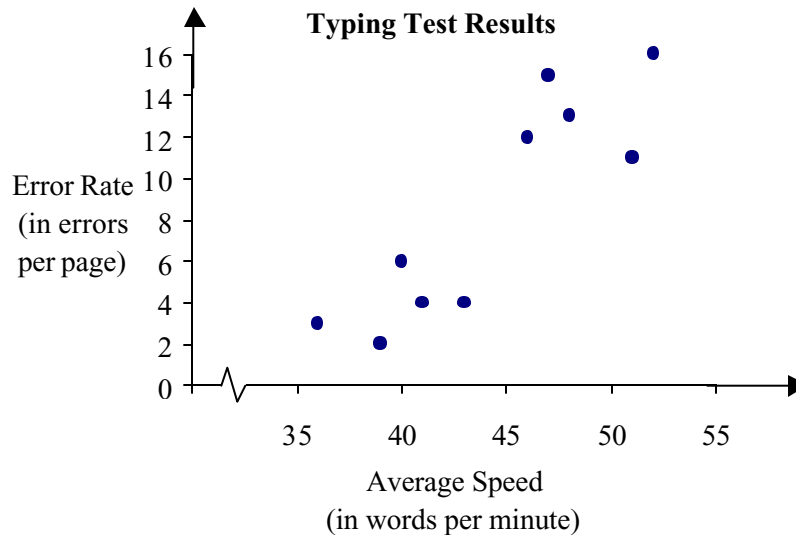
#37

STUDENT RESPONSES AND SCORING

A **4-point** response includes an appropriate graph of the data (Part A) and a description of the relationship between Mark's average typing speed and error rate (Part B).

Part A:

Exemplar:



Part B: As Mark's average typing speed increases, his error rate increases.

The 4 points are broken down as follows

3 points total are possible for graph (Part A)

3 points = graph complete, correct, and labeled (title not needed)

2 points = graph complete, but labeling and/or data points are incorrect

1 point = labeling and data are incomplete

NOTES:

A response which gives a *bar* or a *line* graph automatically loses one point and may receive a maximum of 2 points total.

2 points are given if it is ranked, graphed, and labelled.

1 point is given if it is not ranked but labeled, not ranked but all data points included, or ranked but with mistakes.

Histograms are not acceptable.

1 point for description (Part B)

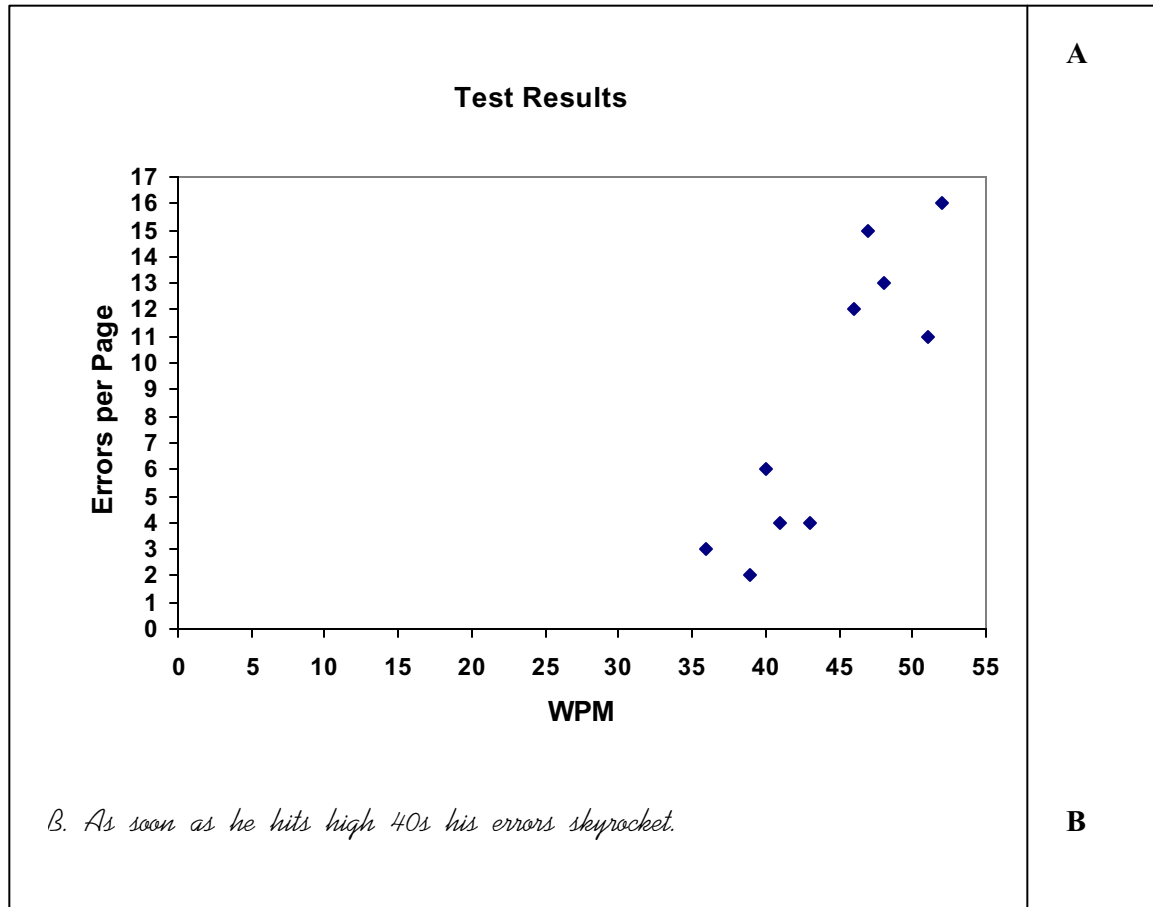
A **3-point** response achieves 3 of the 4 possible points.

A **2-point** response achieves 2 of the 4 possible points.

A **1-point** response achieves 1 of the 4 possible points.

A **0-point** response shows no mathematical understanding of the problem.

Student Response (4 points)



Score 4

This is a 4-point response because

- (A) Response includes an appropriate graph of the data, and a description of the relationship between average typing speed and error rate. This graph includes appropriate scales and labels for each axis and a title.
- (B) Description includes the sharp increase in the error rate between 43 and 46 words per minute.

A.

A scatter plot with 'Average Speed' on the x-axis and 'Error Rate' on the y-axis. The x-axis ranges from 0 to 55 with major ticks every 5 units. The y-axis ranges from 0 to 16 with major ticks every 1 unit. There are 11 data points plotted as blue diamonds. The points are approximately at (36, 3), (39, 2), (40, 6), (41, 4), (43, 4), (46, 12), (47, 15), (48, 13), (51, 11), and (52, 16). The points show a general upward trend, indicating that as average speed increases, the error rate also tends to increase.

Average Speed	Error Rate
36	3
39	2
40	6
41	4
43	4
46	12
47	15
48	13
51	11
52	16

B. As Mark's speed increases, the error rate increases. As you can see from the graph, as speed goes up so does the errors.

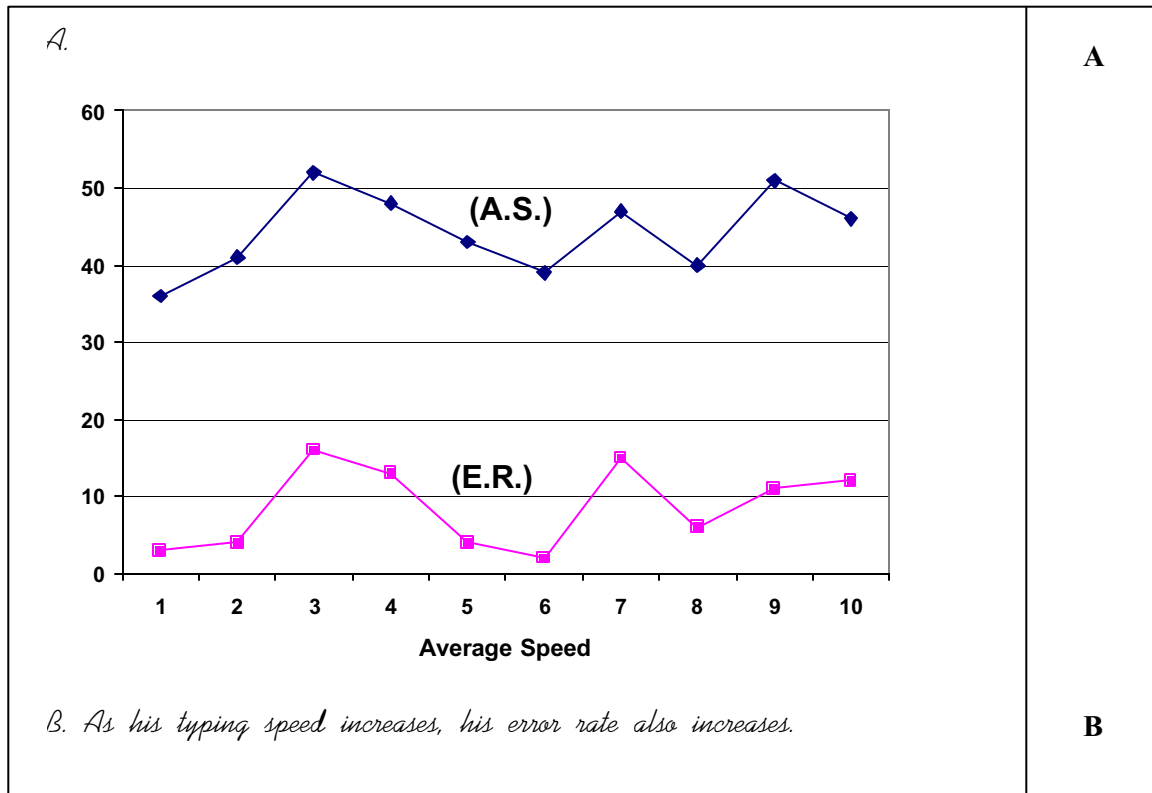
A

B

This is a 3-point response because

- 66

Student Response (2 points)

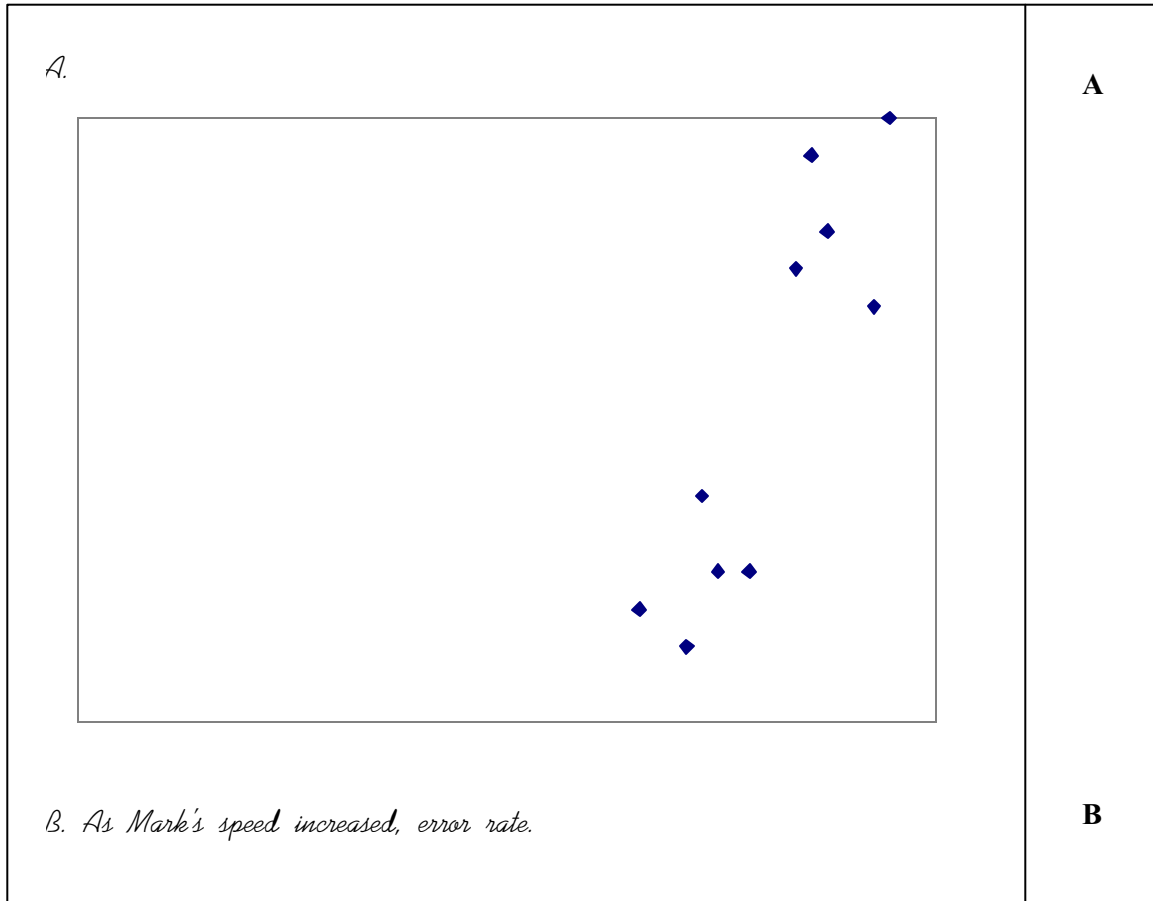


Score 2

This is a 2-point response because

- (A) The line graph automatically loses one point. The labeling is incomplete. There is no title and no label for the y-axis. The response earns 1 point for Part A.
- (B) The description earns 1 point. This response earns 2 of the 4 possible points for a score of 2.

Student Response (1 point)



Score 1

This is a 1-point response because

- (A) An appropriate graph is attempted, but labeling and data are incomplete. The student earns a total of 1 point for the graph.
- (B) The description is incomplete and is awarded no points. The response earns 1 of the 4 possible points for a score of 1.

Student Response (0 points)

<i>B. slower more errors</i>	B
------------------------------	----------

Score 0

This is a 0-point response because

- (A) A graph is not attempted.
- (B) The response has an incorrect conclusion from the data and contains no supporting work.
This response earns 0 points.

ANSWER GRID

Item	Benchmark	Answer
1	2.2.5	B
2	1.1.3	A
3	3.2.1	C
4	3.2.*	A
5	2.1.6	D
6	6.1.3	B
7	5.2.4	C
8	6.2.1	C
9	1.1.5	B
10	4.1.1	B
11	5.2.3	D
12	3.2.2	C
13	5.1.2	A
14	3.2.3	D
15	1.1.5	OE
16	5.2.4	OE
17	2.1.7	OE
18	3.1.4	D
19	4.3.1	A
20	1.1.4	C
21	2.3.2	C
22	1.2.1	B
23	5.1.1	C
24	3.3.4	D
25	6.1.5	B
26	5.2.4	D
27	2.1.7	B
28	6.2.1	D
29	1.1.2	D
30	1.2.3	B
31	5.2.1	C
32	2.3.3	A
33	4.3.2	C
34	2.1.7	A
35	3.2.1	D
36	6.1.3	OE
37	3.3.3	OE

* This item assesses both the strand and the standard, but does not match one specific benchmark.

PART FOUR: RESOURCES AND ACKNOWLEDGMENTS

GLOSSARY

Analytic Scoring: The awarding of separate scores for different traits or dimensions of a student's work.

Assessment: The gathering of evidence to judge a student's demonstration of learning. Aids educational decision making by securing valid and reliable information to indicate whether students have learned what is expected.

Benchmark: A statement of what students are expected to learn at various developmental levels (i.e., elementary school, middle school, and high school) to indicate progress made toward meeting a content standard.

Content Standard: A statement indicating what students are expected to know and be able to do by the time they graduate.

Context: General topics expected to be included in the content of a K–12 curriculum that serve as a basis for test items. Context can make the value of mathematics evident.

Curriculum: A coherent plan for a designated period of time specifying the content that students are expected to understand and apply. Normally includes standards, benchmarks, and a sequence of content that serve as the basis for instruction and assessment.

Holistic Scoring: Scoring based on an overall impression of a work rather than on an accumulation of points.

Instruction: The decisions and actions of teachers before, during, and after teaching to increase the opportunities for student learning.

Performance Standards: A description of performance levels based on mathematics content standards and benchmarks. Performance standards serve as a bridge between what is taught and what is tested.

Prompt: Information presented in a test item that activates prior knowledge and requires analysis in order for a student to respond.

Scoring Rubric: A scoring rubric is a tool for evaluating student performance on an assessment task. Includes a set of criteria used to determine the level of a student's performance.

Subject Area: A body of content derived from related disciplines and organized for curriculum.

Strand: A category for classifying the content standards of a subject area curriculum. The mathematics content standards are categorized into six strands: patterns, relationships and functions, geometry and measurement, data analysis and statistics, number sense and numeration, numerical and algebraic operations and analytical thinking, probability and discrete mathematics.

MATHEMATICS EDUCATIONAL RESOURCES

Following are web addresses that contain mathematics educational resources consistent with the Michigan Curriculum Framework.

The Michigan Department of Education at <www.mde.state.mi.us> The state curriculum framework and resources associated with the framework.

The Michigan Department of Treasury – Merit Board at <www.meritaward.state.mi.us> The site for MEAP including state assessment data.

The Michigan Council of Teachers of Mathematics (MCTM) at <mictm.org> The state’s professional organization for mathematics education.

The National Council of Teachers of Mathematics (NCTM) at <nctm.org> The nation’s professional organization for mathematics education. This site also contains an electronic version of the national mathematics standards.

Macomb Intermediate School District at <<http://www.macomb.k12.mi.us/>> A leading intermediate school district in the development of mathematics education materials.

The Eisenhower National Clearinghouse for Mathematics, Science, and Technology (ENC) at <enc.org> Our national mathematics education clearinghouse.

The North Central Regional Educational Laboratory (NCREL) houses the ***Midwest Mathematics and Science Consortia*** (a USDOE funded program) at <www.ncrel.org/msc/msc.htm> Our regional resource for mathematics, science, and technology education.

The Third International Mathematics and Science Study (TIMSS) at <<http://timss.bc.edu>> The latest, and most comprehensive, study in the status of international mathematics education.

The National Assessment of Educational Progress (NAEP) at <<http://nces.ed.gov/nationsreportcard/site/home.asp>> The national assessment and the nations’ report card for mathematics education.

The NSF-funded comprehensive instructional materials projects’ Implementation Centers:

Elementary Instructional Materials: The ARC Center at <www.comap.com/arc>

Middle School Instructional Materials: The Show-Me Center at <showmecenter.missouri.edu>

High School Instructional Materials: COMPASS at <www.ithaca.edu/compass>

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MEAP HIGH SCHOOL MATHEMATICS TEST

Reference Sheet

(December 2001)

Use this information as needed to answer questions on the Mathematics HST.

Miscellaneous

Distance = rate \times time

Interest = principal \times rate \times time

Compound

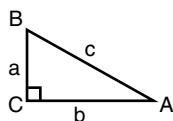
Amount, $A = P(1 + r)^n$, where

P = principal, r = annual rate, n = time

Circumference of a circle = $\pi d = 2\pi r$

$$\pi \approx 3.14 \approx \frac{22}{7}$$

Triangles



$$a^2 + b^2 = c^2$$

Examples:

$$3^2 + 4^2 = 5^2$$

$$5^2 + 12^2 = 13^2$$

Algebra

Straight Line: $y = mx + b$

For points (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Quadratic Formula:

If $ax^2 + bx + c = 0$, $a \neq 0$, then

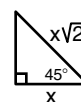
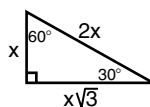
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$



Sequences/Series

Arithmetic Sequence: $a_n = a_1 + (n - 1)d$

Geometric Series: $g_n = g_1 r^{n-1}$

$$S_n = \frac{g(1 - r^n)}{1 - r}$$

Probability

$${}_nC_r = C(n, r) = \frac{n!}{r!(n - r)!}$$

$${}_nP_r = P(n, r) = \frac{n!}{(n - r)!}$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$,
if A and B are two events

$P(A \cup B) = P(A) + P(B)$,
if A and B are mutually exclusive

$P(A \cap B) = P(A) \times P(B)$,
for independent events

Area



Triangle:

$$A = \frac{1}{2}(\text{base}) \times \text{height}$$



Rectangle:

$$A = \text{base} \times \text{height}$$



Trapezoid:

$$A = \frac{1}{2}(\text{sum of the bases}) \times \text{height}$$



Parallelogram:

$$A = \text{base} \times \text{height}$$



Circle:

$$A = \pi r^2$$



Regular Polygon:

$$A = \frac{1}{2}(a) \times \text{perimeter}$$

(Continued on the back)

Total Surface Area

Cone: $SA = \frac{1}{2}(\text{circumference of base}) \times (\text{slant height}) + \pi r^2$

Pyramid: $SA = (\text{perimeter of base}) \times (\text{slant height}) + \text{area of the base}$

Sphere: $SA = 4\pi r^2$

Prism: $SA = \text{sum of the area of the faces}$

Cylinder: $SA = \text{circumference of the base} \times \text{height} + 2\pi r^2$

Cube: $SA = 6 \times (\text{length of edge})^2$



$V = \frac{1}{3}\pi r^2 \times \text{height}$



$V = \frac{1}{3} \text{area of the base} \times \text{height}$



$V = \frac{4}{3}\pi r^3$



$V = \text{area of the base} \times \text{height}$



$V = \pi r^2 \times \text{height}$



$V = (\text{length of edge})^3$

Trigonometry

Angle (Degrees)	Sin	Cos	Tan
0	.0000	1.0000	.0000
1	.0175	.9998	.0175
2	.0349	.9994	.0349
3	.0523	.9986	.0524
4	.0698	.9976	.0699
5	.0872	.9962	.0875
6	.1045	.9945	.1051
7	.1219	.9925	.1228
8	.1392	.9903	.1405
9	.1564	.9877	.1584
10	.1736	.9848	.1763
11	.1908	.9816	.1944
12	.2079	.9781	.2126
13	.2250	.9744	.2309
14	.2419	.9703	.2493
15	.2588	.9659	.2679
16	.2756	.9613	.2867
17	.2924	.9563	.3057
18	.3090	.9511	.3249
19	.3256	.9455	.3443
20	.3420	.9397	.3640
21	.3584	.9336	.3839
22	.3746	.9272	.4040
23	.3907	.9205	.4245
24	.4067	.9135	.4452
25	.4226	.9063	.4663
26	.4384	.8988	.4877
27	.4540	.8910	.5095
28	.4695	.8829	.5317
29	.4848	.8746	.5543
30	.5000	.8660	.5774
31	.5150	.8572	.6009
32	.5299	.8480	.6249
33	.5446	.8387	.6494
34	.5592	.8290	.6745
35	.5736	.8192	.7002
36	.5878	.8090	.7265
37	.6018	.7986	.7536
38	.6157	.7880	.7813
39	.6293	.7771	.8098
40	.6428	.7660	.8391
41	.6561	.7547	.8693
42	.6691	.7431	.9004
43	.6820	.7314	.9325
44	.6947	.7193	.9657
45	.7071	.7071	1.0000

Angle (Degrees)	Sin	Cos	Tan
46	.7193	.6947	1.0355
47	.7314	.6820	1.0724
48	.7431	.6691	1.1106
49	.7547	.6561	1.1504
50	.7660	.6428	1.1918
51	.7771	.6293	1.2349
52	.7880	.6157	1.2799
53	.7986	.6018	1.3207
54	.8090	.5878	1.3764
55	.8192	.5736	1.4281
56	.8290	.5592	1.4826
57	.8387	.5446	1.5399
58	.8480	.5299	1.6003
59	.8572	.5150	1.6643
60	.8660	.5000	1.7321
61	.8746	.4848	1.8040
62	.8829	.4695	1.8807
63	.8910	.4540	1.9626
64	.8988	.4384	2.0503
65	.9063	.4226	2.1445
66	.9135	.4067	2.2460
67	.9205	.3907	2.3559
68	.9272	.3746	2.4751
69	.9336	.3584	2.6051
70	.9397	.3420	2.7475
71	.9455	.3256	2.9042
72	.9511	.3090	3.0777
73	.9563	.2924	3.2709
74	.9613	.2756	3.4874
75	.9659	.2588	3.7321
76	.9703	.2419	4.0108
77	.9744	.2250	4.3315
78	.9781	.2079	4.7046
79	.9816	.1908	5.1446
80	.9848	.1736	5.6713
81	.9877	.1564	6.3138
82	.9903	.1392	7.1154
83	.9925	.1219	8.1443
84	.9945	.1045	9.5144
85	.9962	.0872	11.430
86	.9976	.0698	14.301
87	.9986	.0523	19.081
88	.9994	.0349	28.636
89	.9998	.0175	57.285
90	1.0000	.0000	undefined